

Hall angle in high- T_c cuprates: Anomalous temperature dependence and anisotropic scattering

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Abstract. The anisotropy of the scattering rates, $1/\tau_{tr} \propto T$ and $1/\tau_H \propto T^2$, implied effectively by the anomalous temperature dependence of the normal-state in-plane Hall angle, $\cot \theta \propto T^2$, observed in the high- T_c layered cuprates is reasoned out to be a natural consequence of the semiclassical Boltzmann transport equation with crossed electric (\mathbf{E}) and magnetic (\mathbf{H}) fields. It is argued that while the scattering rate $1/\tau_{tr}$ describes the longitudinal relaxation of the dipolar E-perturbations to the circular zero-field reference distribution function which is known to correspond to a non-Fermi liquid with $1/\tau_{tr} \propto T$, the scattering rate $1/\tau_H$ describes the transverse relaxation of the H-perturbations to the E-induced shifted reference distribution which is Fermi-liquid-like giving $1/\tau_H \propto T^2$. Incorporation of impurity scattering gives $\cot \theta_H = aT^2 + b$ in agreement with the observed temperature dependence.

The temperature dependence of the normal-state in-plane Hall angle $\theta_H(T)$ observed in the layered cuprate superconductors is known¹⁻⁷ to be anomalous in that $\cot \theta_H(T) = a + bT^2$ while the in-plane resistivity $\rho_{ab}(T)$ is T-linear and the Hall coefficient $R_H \propto 1/T$. This is in clear violation of the Kohler scaling rule,^{6,7} known to be valid generally for conventional metals, that assumes isotropic scattering rates, i.e., $\tau_{tr} = \tau_H$, defined with respect to the Hall geometry. Here τ_{tr} is the usual longitudinal relaxation time for

transport parallel to the applied in-plane electric field (\mathbf{E}) while τ_H is the transverse relaxation time for transport perpendicular to \mathbf{E} and the out-of-plane magnetic field (\mathbf{H}). Further, it has been found that (i) the anomaly is generic to high- T_c layered cuprates, (ii) it is pronounced for optimally doped cuprates with T-linear $\rho_{ab}(T)$, and (iii) it gets weaker for over doped samples. Also, no such anomaly is seen in the case of an out-of-plane Hall current.³

Semi-phenomenological approaches to resolving the Hall-angle puzzle have invoked anisotropic scattering rate, $\tau_H \neq \tau_{tr}$, following the proposal due originally to Anderson⁸ that these two distinct scattering rates arise naturally from the spin-charge separation of carriers in the normal state of the layered cuprates. In this note we have argued out, in generality, the observed anisotropy, $\tau_H \neq \tau_{tr}$, basing on a re-interpretation of the structure of the Boltzmann transport equation for crossed \mathbf{E} - and \mathbf{H} -fields without any microscopic particularity. The basic idea is that while the longitudinal response to an electric field \mathbf{E} involves relaxation of a dipolar deformation to the circular reference phase-space distribution, given to be a non-Fermi liquid with T-linear $\rho_{ab}(T)$ implying $\tau_{tr}^{-1} \propto T$, the incremental transverse (Hall) response to the add-on cross field (\mathbf{H}) involves relaxation to a reference state which is now a rotated dipolar sub-distribution. The latter may be viewed as a dilute Fermi-system *per se*, but one with an *excluded* circular phase-space, that makes it a normal-Fermi liquid with the usual relaxation rate $\tau_H^{-1} \propto T^2$.

Our argument can be appreciated best by reference to the structure of the Boltzmann equation for the velocity distribution function under the perturbing crossed \mathbf{E} - and \mathbf{H} -fields. As is well known, the Hall effect is a

multiplicative response where the \mathbf{E} -field produces a dipolar deformation of the unperturbed circular distribution function $f_{o,o}(\mathbf{v})$, and the \mathbf{H} -field response consists in the rotation of this dipolar deformation by an angle, the Hall angle θ_H .

Let $f_{o,o}(\mathbf{v})$ be the distribution function for $\mathbf{E} = 0 = \mathbf{H}$ and $\delta f_{E,o}(\mathbf{v})$ be the dipolar deformation generated by the in-plane electric field \mathbf{E} . Then (in obvious notation⁶)

$$\delta f_{E,o}(\mathbf{v}) = -\tau_{tr} e \mathbf{E} \cdot \mathbf{v} \frac{\partial f_{o,o}}{\partial \epsilon} \quad (1)$$

Here τ_{tr}^{-1} denotes the relaxation rate of deformed distribution $f_{E,o}(\mathbf{v})$ towards the reference circular distribution $f_{o,o}(\mathbf{v})$. Now, consider the effect of the crossed out-of-plane-field \mathbf{H} . This field acts on the dipolar deformation $\delta f_{E,o}(\mathbf{v})$ and generates a rotationally re-deformed distribution $\delta f_{E,H}(\mathbf{v})$. (The action of \mathbf{H} on the original circular reference distribution $f_{o,o}(\mathbf{v})$ generates (quantum-mechanically) only a diamagnetic response which is not relevant here.) The question now is how $\delta f_{E,H}(\mathbf{v})$ is to relax. The first point to note is that $\delta f_{E,H}(\mathbf{v})$ has to relax to the dipolar reference distribution $\delta f_{E,o}(\mathbf{v})$. This reference state characterized by the dipolar deformation $\delta f_{E,o}(\mathbf{v})$ may now be treated as a fermionic sub-system. The latter is certainly dilute (for low-enough E-field). Also, the rotational relaxation $\delta f_{E,H}(\mathbf{v}) \rightarrow \delta f_{E,o}(\mathbf{v})$ is not affected by the background $f_{o,o}(\mathbf{v})$ which merely provides an *excluded phase-space*. Thus, the relaxation $\delta f_{E,H}(\mathbf{v}) \rightarrow \delta f_{E,o}(\mathbf{v})$ is essentially characteristic of a normal Fermi liquid. We must, therefore, have (up to orders linear in \mathbf{E} & \mathbf{H})

$$\delta f_{E,H}(\mathbf{v}) = \delta f_{E,o}(\mathbf{v}) + H \tau_H \frac{e}{c} \mathbf{v} \times \hat{H} \frac{\partial v}{\hbar \partial \mathbf{k}} \frac{\partial}{\partial \mathbf{v}} \delta f_{E,o}(\mathbf{v}) \quad (2)$$

with \hat{H} a unit vector along \mathbf{H} , and $\tau_H^{-1} \propto T^2$ as for a normal Fermi liquid

without impurity scattering.

From Eqs. (1) and (2), we at once have for the Hall angle θ_H :

$$\cot \theta_H \propto \frac{1}{\tau_H} \quad (3)$$

Now, for an impure normal Fermi-liquid $\tau_H^{-1} = \alpha T^2 + \beta$, where β denotes the effect of impurity scattering. Hence,

$$\cot \theta_H = aT^2 + b \quad (4)$$

as indeed observed by Chien *et al.*¹ in Zn-doped YBCO single crystals, and subsequently reported more widely.²⁻⁷

The above argument is consistent with the absence of anomaly when the Hall current is out-of-plane (i.e., along the C-axis) with the crossed-fields in-plane. This is because the C-axis transport has been shown to be controlled intrinsically by the in-plane transport in the normal state, and hence there is a single relaxation time $\tau_{tr} \propto T^{-1}$ consistent with the T-linear in-plane resistivity.⁹⁻¹¹

We would like to conclude with the following comments. The main point of our argument is that while the system may be a strongly correlated one with non-Fermi liquid characteristics, e.g., the T-linear in-plane resistivity, a small deformation (in the sense of distribution function) of the system when probed appropriately may behave differently, and in particular as a normal Fermi-liquid. This notion is, of course, somewhat familiar in terms of the idea of the electron- or the hole-pockets of a complex Fermi-surface representing sub-sets of carriers with different characteristics, e.g., effective masses, etc. In our case of the Hall angle, the electric field prepares the small deformation (sub-set of carriers) and the magnetic-field probes it. It

should be possible to extend this argument to other, possibly multipolar, deformations.

References

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